American University of Beirut Department of Electrical and Computer Engineering

EECE 440 Signals and Systems Homework 2: Due July 18, 2006

Part I Problem 1

Consider a unity feedback linear time-invariant control system whose input is r(t) and output is c(t). The forward transfer function of this system is given by:

$$G(S) = \frac{20(S+1)}{S(S+2)(S+4)}$$

Represent this system in a state variable form by determining the state equation and the output equation.

Problem 2

The open-loop transfer function of a unity feedback control system is given by:

$$A(S) = \frac{(S+1)}{2S^4 + 4S^3 + 4S^2 + aS + 1}$$

Determine the stability of the system (state the reason) for the following two cases when a=0 and a=8.

Problem 3

The dynamic equations of a linear time-invariant system whose output is y(t) and input

$$\begin{bmatrix} \mathbf{X}_1'(t) \\ \mathbf{X}_2'(t) \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1(t) \\ \mathbf{X}_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{r}(t), \qquad \mathbf{y}(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1(t) \\ \mathbf{X}_2(t) \end{bmatrix}$$

r(t) are given by:

a. Determine the stability of the above system.

b. Determine the gains (f1, f2) of the input

$$\mathbf{r}(\mathbf{t}) = \begin{bmatrix} \mathbf{f} 1 & \mathbf{f} 2 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1(\mathbf{t}) \\ \mathbf{X}_2(\mathbf{t}) \end{bmatrix}$$

in order to force the closed-loop system poles in the desired locations in the s-plane (-5 and -3).

Problem 4

A temperature control system is shown below. Sensor 1 measures the temperature, and this sensor is slow compared to the rate at which the temperature can change. Sensor 2 is called the rate sensor.



- 1. Find the values of K that will result in the system being oscillatory.
- 2. With K equal to the value found in (1), determine the period of oscillation.
- 3. Determine the range of K for stability.
- 4. Determine the steady-state error for a constant input provided that the system is stable.

Problem 5

Consider a linear control system whose system transfer function is given by:

$$\frac{\mathbf{C}(\mathbf{s})}{\mathbf{R}(\mathbf{s})} = \frac{\mathbf{K}}{\mathbf{s}+1}$$

- a. For K=10, find the unit step response.
- b. Find the time constant. The time constant T is defined as the time T that satisfies the following equation: $c(T) c(\infty) = 0.368[c(0) c(\infty)]$.
- c. Can you have another definition or meaning of the time constant?
- d. For K=10, close the loop with a constant feedback gain H. Design H so that the unitstep response reaches (and never decreases below) the 63% of its final value within the first 0.5 sec.

Problem 6

The open-loop transfer function of a unity feedback control system is given by:

$$G(s) = \frac{\frac{10}{z}(s+z)}{(s+1)(s+10)}$$

Determine z so that the step response percentage overshoot is 0.1.%

Problem 7

The open-loop transfer function of a unity feedback control system is given by:

$$G(s) = \frac{3s+10}{s^3+2s^2+s}$$

Determine the steady-state value (final value) of the output signal for a unit-step input.

Problem 8

The closed-loop transfer function of a control system is given by:

$$\frac{C(s)}{R(s)} = \frac{s+1}{s^6 + s^5 + 5s^4 + 5s^3 + 7s^2 + 6s + 3}$$

- **1.** Construct the RH table of the above system <u>using the \in -method</u>.
- 2. Choose $\in > 0$ and determine the number of poles of the closed-loop system in each half of the complex plane.
- 3. Choose $\in < 0$ and determine the number of poles of the closed-loop system in each half of the complex plane.
- 4. Comment on the results obtained in parts 2 and 3.
- 5. Using another procedure, determine the number of poles of the system in each half of the complex plane.

Part 2: Matlab Project

1. Stability using MATLAB.

You are requested to determine a procedure to check on the stability of a LTI continuous using MATLAB. Give procedure and a minimum of two examples.

2. State Variable Analysis using MATLAB.

Determine a MATLAB procedure for representing a system defined by its transfer function in a State-variable representation. Give a minimum of two examples.